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Deriving and applying Newton's PQRST formula with pre-service phyiscs teachers

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ABSTRACT

One of Newton's greatest scientific achievements was to show that Kepler's first law follows from the assumption of an inverse-square central force. Despite its importance, this connection is rarely taught in physics courses at introductory level due to the mathematical complexities involved in the proof. A possible didactic solution to this problem is to focus on a conceptual understanding of Proposition VI of Newton's Principia. In this paper, we report a study conducted with the goal of teaching pre-service physics teachers key aspects of Proporsition VI, as well as its application to determine the force law, given *the* orbit shape and the sun's position. Our findings consist of students' interpretations and difficulties when trying to understand Newton's original reasoning.

Keywords: *inverse-square central force, Newton's original reasoning*

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INTRODUCTION

In December 2016, Isaac Newton's Principia Mathematica made the news by becoming the most expensive science book ever sold. It is hard to overestimate the importance of this work for the development of modern science, although it is also fair to say that this book is more revered than read (Brackenridge, 1996). One episode that motivated the writing of the *Principia* is a visit paid by Edward Halley to Issac Newton in August 1684 (Westfall, 1983). Together with other members of the royal society, including Robert Hooke and Christoper Wren, Halley was seeking for an explanation for planetary motion (Provost, 2009). More spesifically, Halley asked Newton which curve would be described by the planets supposing the force of attraction towards the sun to be reciprocal to the square of their distance from it (Westfall, R.S, 1983).

Newton's prompt answer was "an ellipse" and a proof was sent to Halley months later in a manuscript titled *De motu corporum in gyrum* (On the motion of bodies in a orbit). In Theorem 3 of this manuscript, Newton expressed the centripetal force as a general geometrical relation between segments, and later applied this theorem to derive different force laws, i.e., F = F (r), for different trajectories (Hsiang,2011). De Motu's Theorem 3 (aka. The PQRST formula) is an absolut gem of the history of science and illustrate essential aspects of Newton's original reasoning. We are confident that there are numerous reasons to teach it,even at high school level. This motivated us to design an intervention to teach the *PQRST* formula to pre-service physics teachers, and investigate how they try to make sense of it (Cushing, 1982).

Newton's PQRST formula express the magniude of centripetal force exerted by the sun on an orbiting planet. In Figure. 1, consider the trajectory of a planet described by a general curve *APQ* (not necessarily an ellipse!) with the sun located at *S* (not necessarily the focus!)(Hect, 2019). At a given instant, the planet is located at *P*. If the Sun were not exerting force at the planet, it would by its nertial tendency, keep moving in a rectiinier and uniform motion in the direction of *PR*. However, because the sun is exerting a central force on the planet, it will end up at point *Q*, i.e., it will be deviated from its inertial trajectory. Newton's aim with Theorem 3 was to express the magnitude of this frce based on relations between segments of Figure 1.

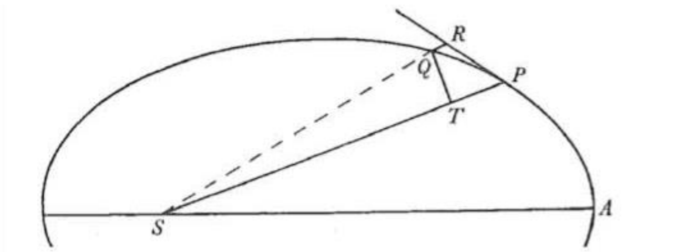

Figure 1. Theorem 3, Force propotional to $F \propto \frac{QR}{SP^2 x QT^2}$

One difficulty to determine the magnitude of the force is that it might change while the planet moves. In order to circumvent this problem, Newton considered that point Q is infinitely close to point *P*, so that it is reasonable to assume that the force does not vary when the planet moves from *P* to *Q* (Lu M, 2017). According to Newton's 2nd law, force is proportional to acceleration, so that the acceleration of the planet will be taken as constant as it moves from *P* to *Q (Naunberg, 2018)*. This is equivalent to a local parabolic approxiamtion, i.e., the motion from *P* to *Q* can be rated as the compposition of uniform motion PR and a uniformly accelerated motion ("free fall") *RQ*.

Motion with constant acceleration was studied extensively by Galileo. In modern terminology, the relation between distance and time for such motion can be expressed by $d = \frac{1}{2}at^2$(Vogt, 1996). Thus, the magnitude of the acceleration is proportional to distance, and inversely proportional to the square of the time (a $\propto \frac{d}{t^2}$). Since F $\propto$ a, we have

$$F \propto \frac{d}{t^2} \quad (1)$$

In Figure 1, the distance travelled in the direction of the force is QR. In order to determine the time, Newton uses a realtion proved in De Motu's Theorem 1, which sates that the line segment connecting the sun and the planet sweeps out equal areas in equal times (Kepler's 2nd law) (Yu, 2010). Another way to formulate this theorem is to say that the time elapsed is proportional to the area swept-out by this line segment, which is approximately equal to the area of the triangle SPQ, since Q is infinitely cose to P. Substituting these consierations in Eq. (1),

$$F \propto \frac{QR}{SP^2 x QT^2} \quad (2)$$

Voilà! This is Newton's PQRST formula. It express the magnitude of the centripetal force in terms of three segments from Figure 1. This formula provides the key to finding the force law, given the orbit shape and the location of the sun. Its derivation was given to pre-service physics teachers in a similar way as presented here. In the first part of this study, we were mainly interested in participants' reasoning and struggles to make sense of this derivation.

Although the PQRST formula provides the key to finding the force law, applying Eq. 2 is far from being trivial. The reason is that as *Q* approaches *P*, both *QR* and *QT* tend to zero, which leads us to the challanges of calculating with infinitesimals (Kavanagh, 2007). The solution involves realizing that although both *QR* and *QT* tend to zero when *Q* approaches *P*, the ratio QR/QT^2 does not. The trick is to use geometrical propertes of the given orbit shape to express this (ultimate) ratio as a function of *SP*, and thus obtain *F = F (r)* (Feynman, 1967).

After having derived the PQRST formula in the De Motu, Newton applies it to solve of three different configurations. In problem 1, the planet's trajectory is circular with the sun located at the circumference, and the force law obtained is $F \propto \frac{1}{r^5}$. In problem 2, the trajectory is an ellipse with the sun is at the center, and the solution is $F \propto r$. Finally, in Problem 3 the trajectory is an ellipse with the sun in one focus, leading to a $F \propto \frac{1}{r^2}$. Thus, Halley's quetion was answered, and the connection between $F \propto \frac{1}{r^2}$ and the elliptical trajectory with the sun at the focus was demonstrated. Figure.2 summarizes the three problems solved at the *De Motu*.

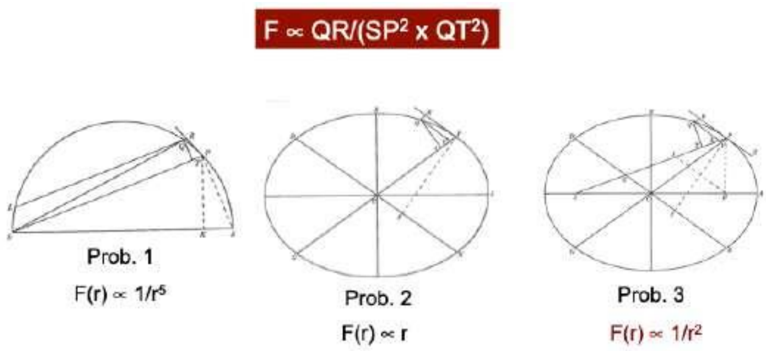

Figure 2. Three problems solved in Newton's De Motu

The solution to these problems involve appying highly complicated geometrical properties, especially in the ellipical case (Brackenridge, 1996), which make them inaccessible for the participants of our study. In order to circumverent this obstacle, and still provide an idea of how the PQRST formula can be used to find the force law, we decided to use an activity proposed by Prentis et al. (3). The activity consist in asking students to draw an orbit (Figure 3), measure the values of the segment QR, QT and SP at different points of the orbit, and use the PQRST formula to estimate the force law, i.e., the dependence of force on the distance between the planet and the sun (F = F(r)) . The process is called "Newton's recipe", and is desccribed by the authors in the following six steps (Prentis, 2007):

> Given only two ingredients – the shape of the orbit and the center of the force – "Newton's recipe" allows one to calculate the relative force at any orbital point. The recipe consists of the following steps:
>
> 1. *The inertial path*: Draw the tangent line to the orbit curve at the point P where the force is to be calculated.
> 2. *The future point:* Locate any future point *Q* on the orbit that is close to the initial point *P*.
> 3. *The deviation line: Draw the line segment from Q to R, where R is a point on the tangent, such that QR (line deviation) is parallel to SP (line of force)*
> 4. The time line*: Draw the line segment from Q to T, where T is about on the radial line SP, such that QT (height of "time triangle" is perpendicular to SP (base of triangle).*
> 5. *The force measure: Measure the shape parameters QR, SP, and QT, and calculate the force measure* $^{QR}/_{(SP\ x\ QT)^2}$.
> 6. *The calculus limit: Repeat steps two five for several future points Q around P to obtain several force measures.*

Take the limit QP of the sequence of force measures to find the exact value of the force measure at P.

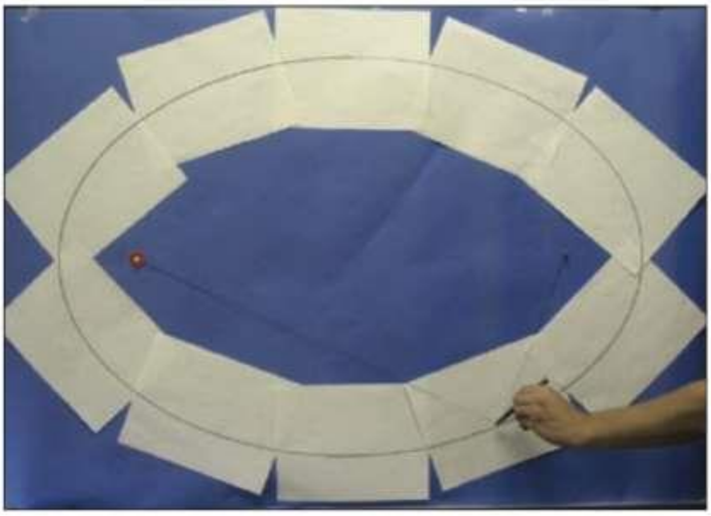

Figure 3. Drawing the orbit to obtain a force aw from applying the PQRST formula (Prentis, 2007, p 23)

The activity described by "Newton's recipe" was given to physics pre-service teachers who participated in our study, and in its second part we focus on their understanding and struggles to follow and interpret these steps.

RESEARCH METHODS

Five pairs of students from the Department of Physics Education of the Sultan Ageng Tirtayasa University in Indonesia were recruited for a semi-structured iterview conducted by the first author (YO). All of them were in their third academic year and had already learned the connection between an inverse square force and an elliptical orbit, both at introductory as well as in more advanced courses on machanics.

This study was conducted in two parts. The first part aimed at probing students' conceptions and struggles with the derivation of Newton's *PQRST* formula, as presented in Section II. The students watched a video made by Gary Rubinstein, which explains the derivation in a pedagogical manner. The students were allowed to watch the video more than once and cloud pause or repeat it whenever they wanted.

During the first interview session, the following three questions were posed to the students, addressing specific conceptual issues of the derivation: i) Why can we consider the motion from R to Q as "free fall", and why can we write $d \propto at^2$? ii) Why can we express forc by $F \propto \frac{d}{t^2}$? iii) Why can $F \propto \frac{QR}{SP^2 x QT^2}$?

The second part of this study focuses on students' conceptions and struggles with the application of Newton's PQRST formula to obtain a force law given the shape of the orbit and the position of the sun. More spesifically, the students were provided with the six steps of the aforementioned "Newton's recipe" and asked to reflect on the application of Newton's PQRST formula.

The second interview session focused on i) Students' general understanding of the role of Newton's PQRST formula; ii) Their understanding and struggles when using "Newton's recipe" to determine the force law for an elliptical orbit with the sun iia) in one focus and iib) in the center of the ellipse, and iii) Students' views on how Newton's

PQRST formula can be used to prove Kepler's 1st law of planetary motion.

In both parts of the study, the students were provided with worksheets. The first part contains the derivation of Newton's PQRST formula based on the video, while the second explains the use of Newton's PQRST formula in proving the connection between the elliptical orbit with the force and the distance. Both worksheets contained questions that guided the interview to the conceptual is sues we intended to investigate.

Due to the lack of previous research on the topic, we chose to conduct an explorative qualitative study. Following a traditional think-aloud protocol, the students were asked to discuss in pairs the questions formulated in the worksheets, as well as things they had difficulties in understadning, while the interviewer would listen carefully and intervene when necessary. Each pair was encouraged to use the whiteboard to register their discussions, which were video recorded.

Our data consist of students' discussions, worksheets, interview protocols, and sketches made by the students on the whiteboard and the worksheets. These data are analyzed thematically, were we look for the students' conceptions and their main struggles with deriving and applying Newton's PQRST formula.

RESULTS AND DISCUSSION

Part 1: Deriving Newton's PQRST formula

1.1 Missing $\frac{1}{2}$ in Galileo's relation d $\propto at^2$

The first question asked to the students was why the formula $d \propto at^2$, referred in the video by "Galileo's realtion", can be used to describe the motion of the planet from R to Q. To answer the question, students need to realize that the motion of the planet from R to Q is due to a constant force is valid since an infinitesimal time has elapsed when the planet moves from P to Q.

The proportionality sign ($\propto$) in the relation between distance and time for a constant acceleration was a great source of confusion to many students (Karam, 2015). Given that for free fall the relation $d = \frac{1}{2}at^2$ is valid, they struggled to understand why the number $\frac{1}{2}$ "disappeared" in Galileo's relation, as exemplified in the following excerpt:

> S5: What I know from the free fall equation is $h = \frac{1}{2}gt^2$, I do not see a number $\frac{1}{2}$ in Galileo's relation. I think that equation is different from the one I know.

This satement represents a confusion expressed by nearly all participants in this study (7/10). It illustrates a difficulty in understanding the meaning of proportionality, which is crucial in Newton's original geometrical reasoning. In sum, these students do not realize that $d \propto at^2$ is a valid statement to be made from $d = \frac{1}{2}gt^2$, since the proportionality constant does not matter. After identifying this misconception, the interviewer intervened with a short explanation of the validity of $d \propto at^2$ from the equation the students were more familiar with (Pospiech 2019).

But even after clarifying the meaning of proportionality, the validity of the assumption of constant acceleration for the motion of the planet from R to Q was questioned by many students (Ding, 2017). In general, the argument of an infinitesimal time was not perceibed as convincing, and most students seemed to have accepted,rather than understood, it.

1.2 From F = ma to $F \propto \frac{d}{t^2}$

The second question of Part 1 addressed the students' ability to determine the magnitude of the central force from the sun. They were expected to combine Newton's second law ($F = ma$) with Galileo's relation ($d \propto at^2$). Furthermore, the students were asked to consider a situation with variable mass m and explain how that would influence the result, as exemplified in the following transcript from a conservation with S6:

Inetrviewer: How could you prove the equation ($F \propto \frac{d}{t^2}$) that determines the magnitude of force?

S6: I think we could start with *F* = *ma*.

Interviewer: Why can you use this formula?

S6: Because the force from the sun is constant and makes the planet move from R to Q. So we could combine these two formulae, then arriving at ($F \propto \frac{d}{t^2}$).

Interviewer: But what about the mass? Why the mass does not appear in that formula?

S6: Maybe the mass is constant here so that we could dismiss it.

Although S6 realizes that both $\frac{1}{2}$ and m are constants, s(he) does not seem to be clear about the difference between an equality and a proportionality, as s(he) uses only the quality sign (see **Figure.4**).

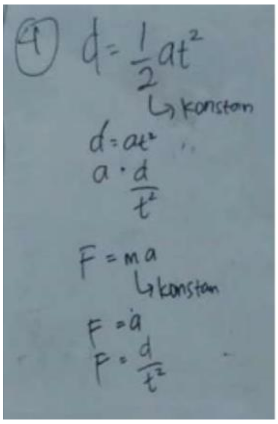

Figure 4. S6's explanation of $F \propto \frac{d}{t^2}$

S4 also struggles to prove that $F \propto \frac{d}{t^2}$. After trying to combine the two formulas, s(he) was confused about the fact that number 2 still appears. S4 argued that the number 2 still appears. S4 argued that the number 2 should be gone, but s(he) does not how (see Figure.5).

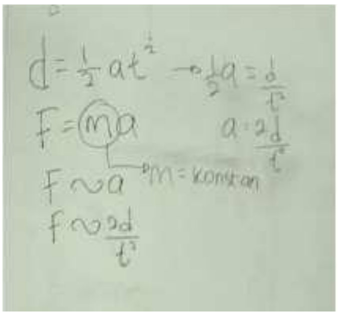

Figure 5. S4'S struggles to explain that $F \propto \frac{d}{t^2}$

Once again, we see the problem with distinguishing an equality from a proportionality relation when S4 writes $F \propto \frac{2d}{t^2}$ and cannot realize that the constant is irrelevant for the statement of proportionality.

Interestingly, one student (S1) associated the formula $F \propto \frac{d}{t^2}$ with an inverse square law $F \propto \frac{1}{r^2}$, but did not provide further justification for her/his reasoning. It is plausible to conjecture that s/he is trying to identify a similarity in the denominators, even though there is no conceptual reason for that ossociation (Karam, 2014). In fact, this represents a misconception, since according to the Newtonian framework, $F \propto \frac{d}{t^2}$ is always valid, whereas $F \propto \frac{1}{r^2}$, is only valid for certain orbits.

1.3 From $F \propto \frac{d}{t^2}$ to $F \propto \frac{QR}{SP^2 XQT^2}$

In order for students to graps the last step in the derivation of Newton's PQRST formula, they need to know the relation between time and the area swept-out by the line segment connecting the Sun and the planet, i.e., Kepler's area law. Since this law states that the line segment will sweep out equal areas in equal times, the variable time (t) in equation $F \propto \frac{d}{t^2}$ can be subtituted by the area (A), which in this case is the area of triangle whose base is SP and height is QT (Figure.1). Furthermore, the distance (d) must be associated with the line segment QR.

All students who participated in this study had also participated in a previous investigation that had as the main focus the relation between a central force nd the area law (Feynman, 1996). Thus, all of them were familiar with Kepler's area law and could understand why times is proportional to the swept-out area A.

However, students' difficulties emerge in defiining the swept-out area. Two of them (S9 and S6) thought that the area should be of a parallogram. S9 argued that the line QR is parallel to the line segment SP, and between these two lines, there is line QT, which perpendicular to both (Figure.1). Thus, for her/him the area should be the product of the line segment SP (base) and the line QT (height). Meanwhile, S6 explained that at the beginnning s9he) thought that the formula, $F \propto \frac{QR}{SP^2 XQT^2}$ is missing the number $\frac{1}{2}$. Therefore, this student concluded that the swept-out area must be parallogram instead of a triangle.

We see here again a lack of deep undestanding of the meaning of proportionality, since they claim that there is a ½ missing in the PQRST formula if the swept-out area should be a triangle. Another difficulty was expressed by S1, who could not identify the height of the triangle Δ SPQ is the line segment QP, instead of QT. When asked to explain his/her reasoning, S1 argued that P would tend to QT when Q → P.

In sum, the main difficulty students ecountered in the first part of this study is to distinguish between an equality and a proportionality relation. In fact, the latter is crucial to understand Newton's original reasoning, which is essentially geometrical. The key issue seems to be that students cannot realize that a constat is irrelevant for the statement of proportionality. This appears to be a robust misconception which is worth furthr investigation. Moreover, the assumption of a constant force since an infinitesimal time has elapsed when the planet moves from P to Q is seen by many as arbitrary.

Part 2: Applying Newton's PQRST formula

2.1 The function of Newton's PQRST formula

The second interview session invetigated sudent's conceptions and struggles in understanding and applying Newton's PQRST formula. The first question in this session adderssed their comprehension of the very role of the PQRST formula. Students were expected to state that this formula allows one to determine the force law, i.e., F = F(r), given the orbit shape and the position of the sun.

This turned out to be quite challenging. Overall, students do not realize the general character of Newton's PQRST formula and often relate it to elliptical orbits and with Newton's gravitational law, as exemplified in following excerpt:

Interviewer: So, what is the role of the PQRST formula?

S1: We can the PQRST formula to determine force or acceleration.

Interviewer: Could you elaborate, how we can use this formula to determine the force?

S1: I think maybe by calculating the gravitational forca of the planet (…)

Interviewer: But, could you tell how we can calcultae the gravitational force by using Newton's PQRST formula?

S1: Mmmm, maybe this PQRST formula is a little bit similar to Newton's gravitational law. SP in PQRST formula is similar to r in Newton's gravitaional law. But I am not sure about the line segments QR and QT.

As we can see, S1 is trying to relate the PQRST formula with Newton's gravitational law, as if they had similar theoretical status. Overall, students' difficulties to make sense of the PQRTS formula are understandable. In fact, the very question of determining a force law, which is different from an inverse square relation, seems rathr unusual to them. Moreover, they are not used to considering other orbit shapes besides an ellipse with the sun in the focus. It appears that they treat Kepler's first law and Newton's gravitational law as evident truths, and do not realize that one can be deduced from the other.

2.2 Applying Newton's recipe to determine F(r) in elliptical orbit for two different positions of the sun

In order to make the role of the PQRST formula more explicit to the students, we decided to ask them to apply "Newton's recipe" in two different cases. The first, for obvious reasons, is an elliptical orbit with the sun in one focus, which should enable the students to obtain the inverse square law. The second is also an elliptical orbit, but with the sun at the center of ellipse. The latter case yields the unexpected result of a force directly proportional to the distance ($F \propto r$), which not only shows that the PQSRT formula has a higher hierarchical status in Newton's theory, but also motivates fruitful discussions about the possibility of a gravitational force that increase with distance. "Newton's recipe" was presented to the students as follows. They started by drawing an ellipse and locating a point P. Then, they had to draw a tangent to the ellipse at point P and locate a (very) nearby point R at this tangent. The next steps was locate point Q, given that the line QR must be parallel to SP, and point T, considering that QT must be the height of the triangle SPQ (see Figure. 6). The next step was to measure three line segments (QR, QT, SP) and use the PQRST formula to calculate the magnitude of the force at point P**.** By repeating the procedure for different points, they would get a set of values for F and correspondent values for r, which were plotted in a table. Finally, a software (e.g. Geogebra) was used to obtain the relationship between force and distance r via regression. The procedure was repeated for the sun at the center and new force law was obtained. The activity was done by each student individually.

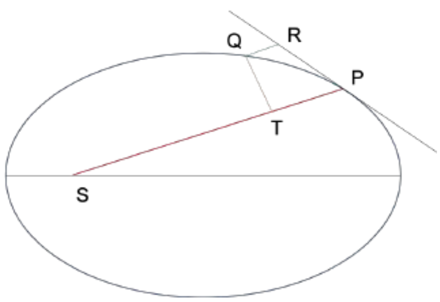

Figure 6. Relevant segments to apply the PQRST formula

There students obtained relationships between F and r that were far from expected $F \propto r^{-2}$. These students (S1, S6 and S8) had some difficulties when drawing, measuring, and defining the force's magnitude. When asked to describe their difficulties, they said that it was hard to draw an ellipse precisely. Furthermore, they mentioned that it was difficult to determine that tangent line at a given point P, and some thought that it is possible to draw several tangent lines for a given point.

Other difficulties arose when they did not draw the line segment QR parallel to SP, and the line segment QT perpendicularly to SP. Figure. 7 (made by S6), illustrate both problems happening in different points. Other three students (S3, S7 and S10) arrived at a relationship between F and r closer to the expected $F \propto r^{-2}$. S3 obtained $F \propto r^{-1.43}$, S7 obtained $F \propto r^{-4.26}$, and S10 got $F \propto r^{-3.38}$. In the interview, these students said that the biggest challenge for them was drawing the line QR, since it is usually a very small segment. Moreover, they expressed difficulties with drawing the height of the triangle correctly. S10 argues that even though the line QT is the height of the triangle, and is perpendicular to the line SP, it does not need to be always inside the ellipse (Figure.8).

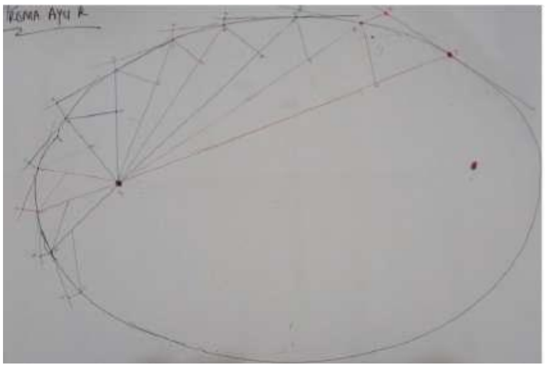

Figure 7. Student draws QR and QT incorectly at some points

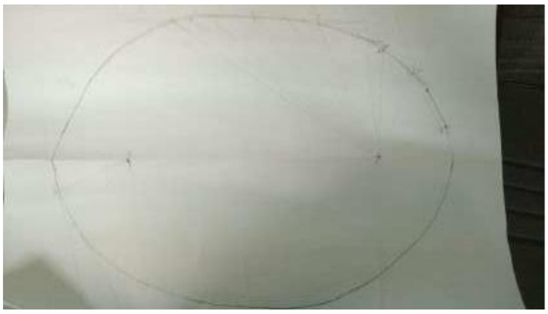

Figure 8.Orbit drawn by S10 where in some points the height of the triangle id "outside" the ellipse.

S2, S5 and S9 obtained results which were pretty close to the expected. S2 got $F \propto r^{-2.44}$, S9 $F \propto r^{-1.89}$ and S5 $F \propto r^{-2}$. All three students mentioned that since the planet moves in an ellipse with the sun located at the focus, the force should be nversely proportional to the square of the distance. This previous expectation might have influenced their result, which should not be expected due to the instrinsic nature of this activity. Figure. 9 shows the diagram drawn by S9, and Figure.10 the table with 10 points (F,r) as well as the curve of best fit.

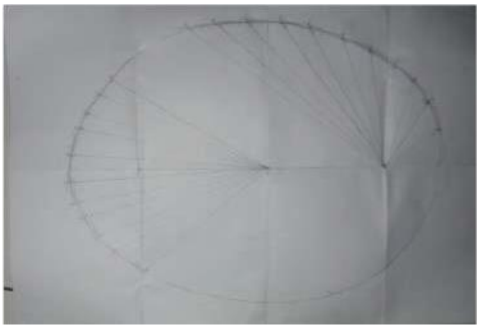

Figure 9. Diagram orbit by S9

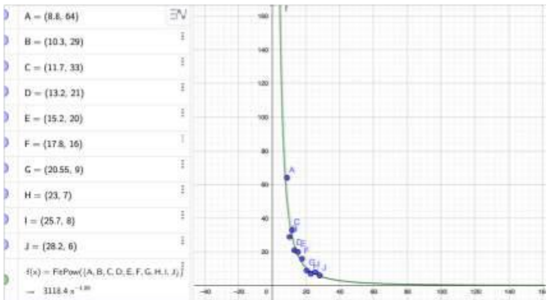

Figure 10. Table with 10 (F,r) points made by S9

The next task was to repeat the procedure for the Sun at the center of the ellipse. This time, they did not know what to expect and could not predict the relationship between force and distance. In fact, they did not even know if it should be different from $F \propto r^{-2}$. The exact result in this case is $F \propto r$, as Newton showed in Problem 2 of his *De Motu*. From the nine students who made the activity, four arrived at a relationship which was far away from $F \propto r$.

An investigating case is found in S8's drawings and tables. Initially, this student decided to take different points P symmetric to the major semi-axis of the ellipse (right-hand side of Figure.11). This led him/her to obtain very similar values for SP (see Figure.12) and made it difficult to draw the tangent to locate the relevant points. Realizing this problem, S8 decided to locate five points with a greater distance between them (left-hand side of Figure.11). Thus, the tangent between one point to another can be distinguished more clearly. When s/he entered the data from these five points (Figure.13) in the software, S8 obtained the relationship $F \propto r^{1.03}$, which is extremely close to the correct theoretical value.

At the end of this activity, students were asked to reflect about the implications to the result obtained for an elliptical orbit with the sun at the center, namely that $F \propto r$,even though many did not arrive at this conclusion from their data.

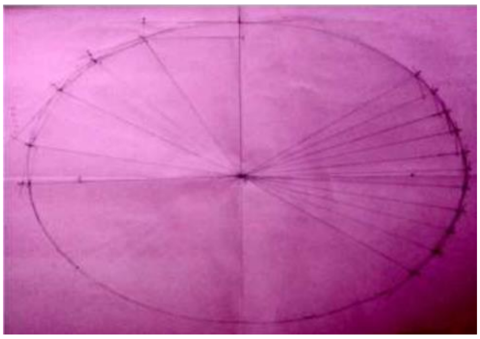

Figure 11. Orbit when the Sun is at the center made by S8.

QR	SP	QT	SP^2	QT^2	$(SP \times QT)^2$	F
0.025 cm	25.5 cm	5.5 cm	552,25 cm	12.25 cm	6.765,06	3.69×10^{-6}
0.03 cm	25 cm	3 cm	625 cm	9 cm	5.625	5.33×10^{-6}
0.03 cm	25.6 cm	2.7 cm	655.36 cm	7.29 cm	4.777,57	6.27×10^{-6}
0.25 cm	25.9 cm	2.7 cm	670.81 cm	7.29 cm	4.890,20	5.11×10^{-6}
0.03 cm	26 cm	2.1 cm	676 cm	4.41 cm	2.981,16	1.0×10^{-6}
0.09 cm	25.7 cm	2 cm	660.49 cm	4 cm	2.641,96	1.5×10^{-6}
0.03 cm	25.3 cm	2.7 cm	640.09 cm	7.29 cm	4.666,25	6.43×10^{-6}
0.021 cm	25 cm	2.5 cm	625 cm	6.25 cm	3.906,25	5.37×10^{-6}
0.04 cm	24.5 cm	1.8 cm	600.25 cm	7.184 cm	4.705,96	8.49×10^{-6}
0.04 cm	23.5 cm	2.7 cm	552.25 cm	7.29 cm	4.025,90	9.93×10^{-6}

Figure 12. Table made by S8 with 10 points (right-hand side of Figure.12)

Guided by the interviewer, the students realized that $F \propto r$ would imply in a gravitational force that increaseas with distance, which is compatible with what they know from Newton's law of gravitaion. Thus students concluded that the real world, the Sun must be at focus, and not at the center of the ellipse.

One of the main goals of our teaching sequence was for the students to realize the epistemological status of Newton's PQRST formula, which is a very general assumption that allows one to obtain the force law, given the orbit shape and the position of the sun. In particular, by applying this formula, they should understand that it is possible to show how Kepler's elliptical law is deeply connected to Newton's gravitational law.

This turned out to be very challenging, understanding that most laws can be derived by general principles. Second, the application of "Newton's recipe" involves several geometrical techniques (e.g. drawing a tangent to a point) that were challenging to many students, and led them to obtain divergent result, which prevented most of them from understanding Newton's PQRST formula.

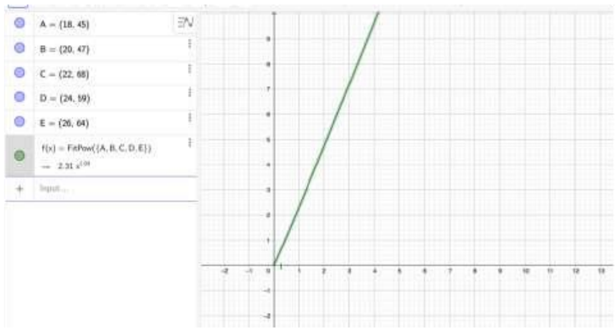

Figure 13. Table made by S8 with 5 points (left-hand side of Fig.11)

CONCLUSIONS

One of Newton's greatest scientific achievements, probably the main responsible for this fame, was to show that Kepler's first law follows from the assumption of a central inverse square force. In a manuscript sent to Edmund Halley known as De Motu, Newton presented this proof by first deriving a more general relation (PQRST formula) and then applying it to specific case of an elliptical orbit with the Sun at the focus.

In this study, a sequence was developed to teach preservice physics teachers the basic of Newton's original reasoning in his De Motu, which was the precursor of celebrated Principia. In the following, we present a summary of the main points learned this study. Our hope is that physics teachers will be motivated to teach Newton's PQRST formula to their students, and learn from our mistakes if they decide to do so. 1) Part of our sequence should enable students to appreciate the deductive structure of physics theories. From the deducted interviews, we realized that they were not used to that kind of reasoning, and had serious difficulties to understand the higher theoretical status of Newton's PQRST formula compared to Kepler's laws or Newton's law universal gravitation. Perhaps this epistemological aspect should be more emphasized in future interventions. 2) Students clearly struggled when reasoning with proportionality, and could not distinguish it from equality. This prevented them from grasping essential arguments of Newton's original geometrical reasoning. A more careful explanation of proportionality reasoning would be needed in future applications of our sequence. 3) The PQRST formula is valid only in the (theoretical) limit $Q \rightarrow P$. This is a crucial point and exemplifies the genesis of Newton's geometrical calculus. This aspect was not emphasized enough, which resulted in students expecting exact results when applying " Newton's recipe". Here we have a good opportunity to discuss the important difference between approximations made with paper and pecil in the drawings, and approximations made with the mind, which we would use more wisely in another application. 4) Applying "Newton's recipe" requires geometrical techniques, like drawing precise ellipses, tangents at given points, perpendicular segment from given lines, etc. The lack of such tehnical skills prevented many participants of our study from obtaining meaningful results for the force law. This technical aspect needs to be seriously in future applications. 5) Newton is famous for having said that he does not make hypotheses (*Hypothesis non figo)*. Although this issue is up for heated debates among historians and philosophers, the PQRST formula does illustrate Newton's position. Contrary to Kepler and Hooke, who had physical reasons/ models to justify their force laws, Newton does not have to make any physical assumption about the nature of gravitaion, and is able to deduce the inverse square force law from pure logical reasoning. We have not highlighted this important aspect in the sequence analysed here, but would do so if we were to teach it again. 6) Comparing the force laws of having the sun at the center and the focus of an ellipse can be extremely instructional. Considering that the eccentries of the orbits in our solar systems are rather small, it is quite counter-intuitive that changing the position of the Sun from the focus of an ellipse can be extremely instructional. Considering that the eccentricities of the orbits in our solar system are rather small, it is quite counter –intuitive that changing the position of the Sun from the focus to the center we have $F \propto r$, which implies a gravitational force that *increases* with distance, contradicting our most basic intuitions about gravity. We would explore the potential of this comparison more if we were to apply the sequence again.

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