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Trends in Mechanical Engineering Research

MODELING AND DYNAMIC ANALYSIS OF VEHICLE SUSPENSION BASED ON STATE SPACE VARIABLE

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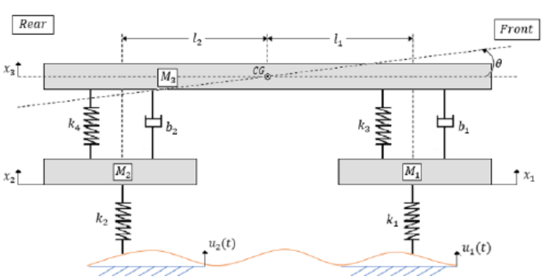
Graphical abstract



Half Car Suspension System Model

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Abstract

The suspension system is a crucial mechanical component in a vehicle, particularly in cars, situated between the vehicle body and the wheels. The suspension system functions to dampen vibrations and prevent their transmission to the vehicle body. The objective of this research is to obtain a system model for a vehicle suspension, which will be subsequently simulated. The simulation results of the developed system model can then be analyzed using MATLAB. After conducting the simulation, the dynamic response generated from the simulation is analyzed. The simulation results for a road bump with a height of 0.2 m and varying speeds of 20 km/h, 30 km/h, and 40 km/h reveal the occurrence of overshoot in the vehicle chassis, measuring 0.032 m, 0.022 m, and 0.017 m, respectively. The settling time for the vehicle chassis occurs successively in 1.59 seconds, 1.33 seconds, and 1.28 seconds. The vehicle tends to return to a stable position more quickly at higher speeds compared to lower speeds. Similarly, the settling time decreases as the speed increases.

Keywords: Modeling, Road Bump, Suspension

Abstrak

Sistem suspensi merupakan salah satu komponen mekanik yang penting dalam suatu kendaraan, utamanya mobil, yang terletak di antara bodi kendaraan dengan roda. Sistem suspensi yang berfungsi untuk menahan getaran yang terjadi agar tidak berpindah pada bodi kendaraan. Tujuan Penelitian kali ini adalah untuk mendapatkan model sistem dari sebuah suspensi kendaraan yang akan disimulasikan. Selanjutnya dapat mensimulasikan hasil permodelan sistem yang telah dibuat melalui MATLAB. Setelah simulasi dilakukan, dapat menanalisa respon dinamis yang dihasilkan dari simulasi yang telah dilakukan. Hasil Simulasi pada *road bump* setinggi 0.2 m dengan kecepatan yang bervariasi yaitu 20 km/h, 30 km/jam, dan 40 km/jam memperlihatkan bahwa terjadinya *Overshoot* pada *Chassis* mobil berturut-turut sebesar 0.032 m, 0.022 m, dan 0,017 m dimana *Settling time* dari *Chassis* mobil tersebut terjadi berturut-turut selama 1.59 detik, 1.33 detik, dan 1.28 detik. Kendaraan cenderung kembali ke posisi tetap lebih cepat dibandingkan dengan kecepatan yang lebih rendah. Sama halnya pada settling time nya semakin cepat ketika kecepatannya lebih tinggi.

Kata kunci: Permodelan, *Road Bump*, Suspensi
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1.0 INTRODUCTION

A model is defined as a logical description of how a system works or how its components respond. The system is a unity composed of components or elements interconnected to facilitate the flow of information, materials or energy. So, system modeling is a simplified form of very complex elements and components, making it easy to understand the required information. [1] The suspension system is a rigid mechanism that is located between chassis (body) with wheels that have the function of absorbing vibrations or shocks (dynamic loads) that occur due to road surface conditions which is uneven. [2] A shock absorber is a part that functions to dampen axial movement of the spring. When a

vehicle hits a roadbump, the vehicle will experiences reflection several times at its natural frequency. [2] Vehicle dynamics is a study that studies movement the entire vehicle and all degrees of freedom of the vehicle system. Every movement has its own speed, acceleration and frequency. Analytical studies of dynamics require a basic suspension model using different degrees of freedom. Dynamic models have one, two or three dimensions Degree of Freedom (DOF). 3 types of models (7 DOF full vehicle model, 4 DOF half vehicle model, and 2 DOF quarter vehicle model). [2] In this research, the dynamic response of the will be simulated using half vehicle suspension system model. Using the force exerted through stiffness, can usually be written in the form of a general state space equation for harmonics input, the vehicle body and wheel connectors can calculated. The parameter used for calculations is tires stiffness, suspension stiffness, suspension dampers, wheel/unsprung mass, and sprung mass.

2.0 SUSPENSION SYSTEM MATHEMATICAL MODELLING

In this study, the physical model of the car used is based on the Avanza Veloz 2022 model. The reference vehicle model employed for the research is as follows:

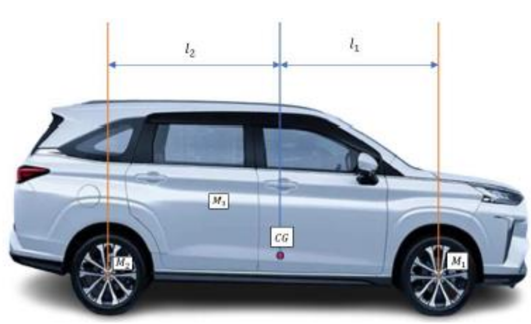

Figure 1. Avanza Veloz 2022 model

After obtaining the physical model of the car, the next step is to create the motion equations for the obtained vehicle system. All masses and components in the half-vehicle system model are depicted in the image below:

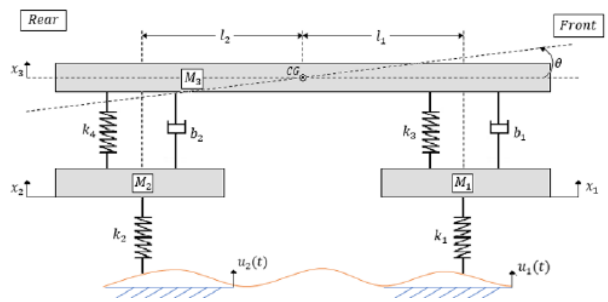

Figure 2. Half Car Suspension System Model

After obtaining the system model, a Free Body Diagram (FBD) of the system can be created using Newton's second law to derive a mathematical model. This model utilizes the equilibrium point as the starting point for the displacement of the Center of Gravity (CG) and the angular displacement of the vehicle body. The Free Body Diagram for each degree of freedom is as follows:

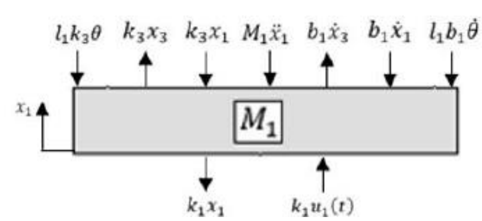

Figure 3. Front Axle FBD

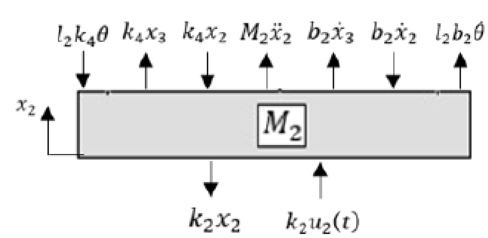

Figure 4. Rear Axle FBD

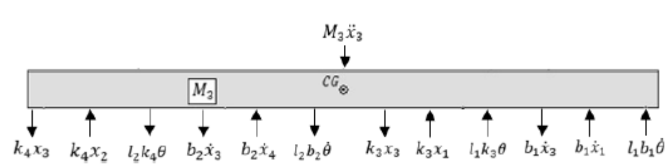

Figure 5. Chasis Translational FBD

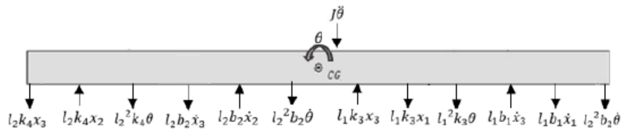

Figure 6. Chasis Rotational FBD

From the Free Body Diagram (FBD) above, mathematical models and the form of the State Space Variables are obtained from the equations derived from the External Forces on the System Model, as well as the Forces and Moments from the Front Axle, Rear Axle, Chassis Translation, and Rotation, as follows:

Front Axle

$$M_1\ddot{x}_1 = k_3x_3 - k_3x_1 - l_1k_3\theta + b_1\dot{x}_3 - b_1\dot{x}_1 - l_1b_1\dot{\theta} - k_1x_1 + k_1u_1(t)$$

State space variable from Front Axle Equation is:

$$\ddot{x}_1 = \frac{1}{M_1}\left[k_3(x_3 - x_1 - l_1\theta) + b_1(\dot{x}_3 - \dot{x}_1 - l_1\dot{\theta}) - k_1(x_1 - u_1)\right]$$

Rear Axle

$$M_2\ddot{x}_2 = k_4x_3 - k_4x_2 + l_2k_4\theta + b_2\dot{x}_3 - b_2\dot{x}_2 + l_2b_2\dot{\theta} - k_2x_2 + k_2u_2(t)$$

State space variable from Rear Axle Equation is:

$$\ddot{x}_2 = \frac{1}{M_2}\left[k_4(x_3 - x_2 + l_2\theta) + b_2(\dot{x}_3 - \dot{x}_2 + l_2\dot{\theta}) - k_2(x_2 - u_2)\right]$$

Chassis (Translational)

$$M_3\ddot{x}_3 = -k_3x_3 + k_3x_1 + l_1k_3\theta - k_4x_3 + k_4x_2 - l_2k_4\theta - b_1\dot{x}_3 + b_1\dot{x}_1 + l_1b_1\dot{\theta} - b_2\dot{x}_3 + b_2\dot{x}_2 + l_2b_2\dot{\theta}$$

State space variable from Chassis (Translational) Equation is:

$$\ddot{x}_3 = \frac{1}{M_3}\left[-k_3(x_3 - x_1 - l_1\theta) - k_4(x_3 - x_2 + l_2\theta) - b_1(\dot{x}_3 - \dot{x}_1 - l_1\dot{\theta}) - b_2(\dot{x}_3 - \dot{x}_2 + l_2\dot{\theta})\right]$$

Chassis (Rotational)

$$J\ddot{\theta} = l_1k_3x_3 - l_1k_3x_1 - l_1{}^2k_3\theta - l_2k_4x_3 + l_2k_4x_2 - l_2{}^2k_4\theta + l_1b_1\dot{x}_3 - l_1b_1\dot{x}_1 - l_1{}^2b_1\dot{\theta} - l_2b_2\dot{x}_3 + l_2b_2\dot{x}_2 - l_2{}^2b_2\dot{\theta}$$

State space variable from Chassis (Rotational) Equation is:

$$\ddot{\theta} = \frac{1}{J}\left[l_1k_3(x_3 - x_1 - l_1\theta) - l_2k_4(x_3 - x_2 + l_2\theta) + l_1b_1(\dot{x}_3 - \dot{x}_1 - l_1\dot{\theta}) - l_2b_2(\dot{x}_3 - \dot{x}_2 + l_2\dot{\theta})\right]$$

To conduct simulations on the half-vehicle suspension system, parameters are required for the mathematical model of the half-vehicle suspension system. The vehicle parameters used are as follows:

Parameters	Symbol	Value
Constants		
Sprung mass of the half vehicle Chassis	M_3	$835\ Kg$
Moment of inertia of the vehicle	J	$418\ Kg.m^2$
Unsprung mass of the front axle	M_1	$28\ Kg$
Unsprung mass of the rear axle	M_2	$28\ Kg$
Stiffness of the front tire material	k_1	$200000\ {}^{N}/_{M}$
Stiffness of the rear tire material	k_2	$180000\ {}^{N}/_{M}$
Spring constant of the front axle	k_3	$25000\ {}^{N}/_{M}$
Spring constant of the rear axle	k_4	$20000\ {}^{N}/_{M}$
Damping coefficient of the front axle	b_1	$1500\ {}^{N.s}/_{M}$
Damping coefficient of the rear axle	b_2	$1200\ {}^{N.s}/_{M}$
Front body length from the CG	l_1	$1.35\ m$
Rear body length from the CG	l_2	$1.40\ m$
State Variables		
Vehicle vertical displacement (Chassis)	x_3	……..
Vehicle rotational movement	θ	……..
Front axle vertical displacement	x_1	……..
Rear axle vertical displacement	x_2	……..

The motion equation of the above mathematical model can be rearranged into matrix form as follows:

$$[M]\ddot{x} + [b]\dot{x} + [k]x = F(u)$$

Where:

$$[M] = \begin{bmatrix} M_3 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_3 \\ \ddot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \ldots$$

$$[b] = \begin{bmatrix} b_1 + b_2 & l_2b_2 - l_1b_1 & -b_1 & -b_2 \\ l_2b_2 - l_1b_1 & b_1l_1{}^2 + b_2l_2{}^2 & l_1b_1 & -l_2b_2 \\ -b_1 & l_1b_1 & b_1 & 0 \\ -b_2 & -l_2b_2 & 0 & b_2 \end{bmatrix}\begin{bmatrix} \dot{x}_3 \\ \dot{\theta} \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \ldots$$

$$[k] = \begin{bmatrix} k_3 + k_4 & l_2k_4 - l_1k_3 & -k_3 & -k_4 \\ l_2k_4 - l_1k_3 & k_3l_1{}^2 + k_4l_2{}^2 & l_1k_3 & -l_2k_4 \\ -k_3 & l_1k_3 & k_3 + k_1 & 0 \\ -k_4 & -l_2k_4 & 0 & k_4 + k_2 \end{bmatrix}\begin{bmatrix} x_3 \\ \theta \\ x_1 \\ x_2 \end{bmatrix} + \ldots$$

$$[F(u)] \ldots = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

State Space Model

$$\dot{x} = Ax + Bu$$

Where:

$$x = [x_1 \quad \dot{x}_1 \quad x_2 \quad \dot{x}_2 \quad x_3 \quad \dot{x}_3 \quad \theta \quad \dot{\theta}]^T, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Where x is the state vector describing the front and rear axle vertical displacements and velocities, translational and rotational displacements of the unsprung mass of the vehicle.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{(k_1+k_3)}{M_1} & -\frac{b_1}{M_1} & \frac{0}{M_1} & \frac{0}{M_1} & \frac{k_3}{M_1} & \frac{b_1}{M_1} & \frac{l_1 k_3}{M_1} & \frac{l_1 b_1}{M_1} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{0}{M_2} & \frac{0}{M_2} & -\frac{(k_4+k_2)}{M_2} & -\frac{b_2}{M_2} & \frac{k_4}{M_2} & \frac{b_2}{M_2} & -\frac{l_2 k_4}{M_2} & -\frac{l_2 b_2}{M_2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_3}{M_3} & \frac{b_1}{M_3} & \frac{k_4}{M_3} & \frac{b_2}{M_3} & -\frac{(k_3+k_4)}{M_3} & -\frac{(b_1+b_2)}{M_3} & \frac{(l_2 k_4 - l_1 k_3)}{M_3} & \frac{(l_2 b_2 - l_1 b_1)}{M_3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{l_1 k_3}{J} & \frac{l_1 b_1}{J} & -\frac{l_2 k_4}{J} & -\frac{l_2 b_2}{J} & \frac{(l_2 k_4 - l_1 k_3)}{J} & \frac{(l_2 b_2 - l_1 b_1)}{J} & -\frac{(l_1{}^2 k_3 + l_1{}^2 k_4)}{J} & \frac{(l_2{}^2 b_2 - l_2{}^2 b_1)}{J} \end{bmatrix} x$$

$$+ \begin{bmatrix} 0 & 0 \\ \frac{k_1}{M_1} & \frac{0}{M_1} \\ 0 & 0 \\ \frac{0}{M_2} & \frac{k_2}{M_2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} U$$

Road Disturbance Input

Road disturbance is an external interference to the road that serves as input ($u(t)$) in the vehicle suspension system. Road disturbances are experienced at the front and rear wheels with the presence of time delay. The calculation of the time delay is as follows:

$$Time\ Delay\ (t_d) = \frac{l_1 + l_2}{v}$$

Where:

l_1 = Jarak dari roda depan ke CG (m)

l_2 = Jarak dari roda belakang ke CG (m)

v = kecepatan kendaraan ($^m/_s$)

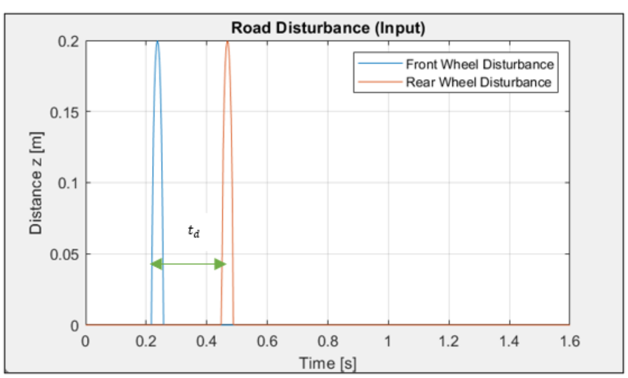

Figure 7. Typical Road Profile

MATLAB Simulation

The simulation is conducted in MATLAB with a total simulation time of 20 seconds. The vehicle travels at a specific speed over a road bump with a height of 0.2 meters, considering speed variations of 20 km/h, 30 km/h, and 40 km/h.

Speed (km/h)	*Speed* (m/s)	*Time delay* (s)
20	5.667	0.485
30	8.333	0.330
40	11.111	0.247

The response of a half-vehicle suspension system can be analyzed through Settling Time and Maximum Overshoot, which are parameters in establishing the effectiveness and robustness of the suspension model.

3.0 SIMULATION RESULTS AND DISCUSSION

In the simulation results obtained from three different speeds against a road disturbance of 0.2m, the following data is generated as follows:

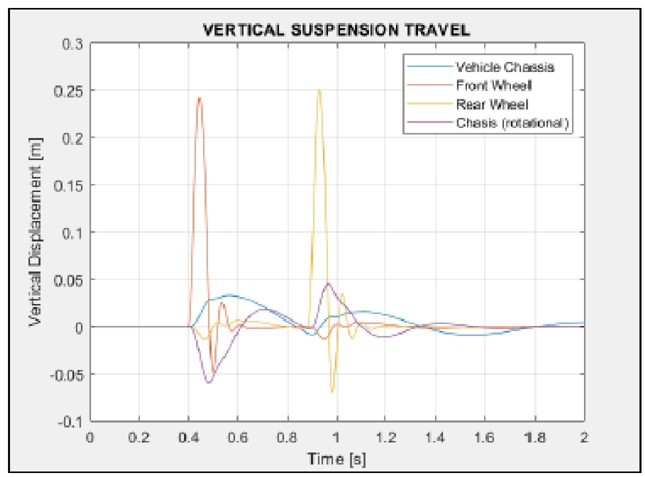

Figure 8. Response at a speed of 20 km/h

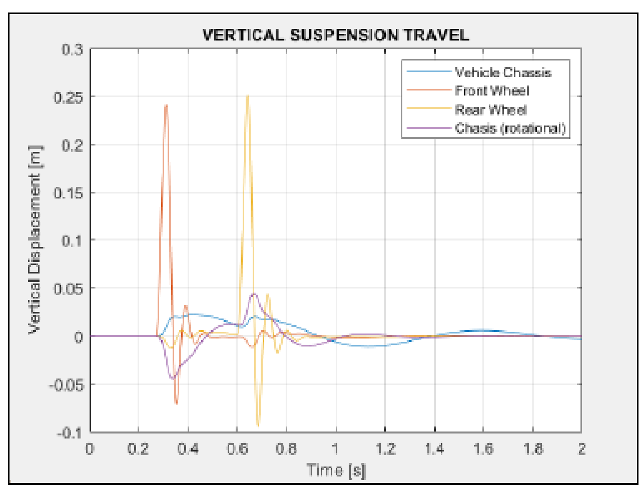

Figure 9. Response at a speed of 30 km/h

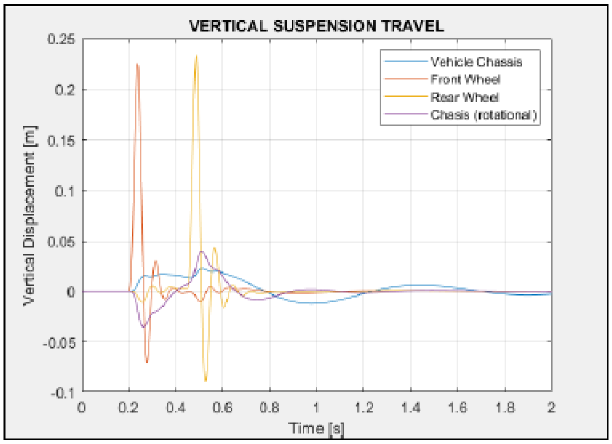

Figure10. Response at a speed of 40 km/h

From the responses above, the following data is obtained:

Displacement	Max. Overshoot (m)			Settling Time (s)		
	20km/h	30km/h	40km/h	20km/h	30km/h	40km/h
Front Wheel	0.243m	0.241m	0.226m	1s	0.89s	0.78s
Rear Wheel	0.253m	0.251m	0.23m	1s	0.89s	0.78s
Chassis	0.032m	0.022m	0.017m	1.59s	1.33s	1.28s
Chassis (rot)	-0.058m	-0.045m	-0.035m	1.59s	1.33s	1.28s

4.0 CONCLUSION

From the conducted simulation and analysis of the simulation results on the suspension system of the half vehicle Avanza Veloz 2022, the following conclusions is the response obtained from the simulation includes the displacement and settling time of the vehicle to return to its original position experienced in the vehicle suspension system at each simulated speed. The maximum overshoot at each point indicates that the vehicle tends to return to a stable position more quickly when the vehicle speed is higher. The time required for the vehicle to return to its original position (settling time) also becomes faster as the vehicle speed increases when interacting with road disturbance. The response occurring in the front and rear suspensions of the vehicle at each speed mutually influences each other. The displacement experienced by the rear part of the vehicle also undergoes changes that result in higher displacement than the front tire displacement due to vibrations in the front suspension system affecting the rear tire.

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