Theta: Journal of Statistics, vol. 1, no. 1, pp. 35-42, 2025

Theta: Journal of Statistics

Journal homepage: www.jurnal.untirta.ac.id/index.php/tjs

Similarity Analysis of the Default Transition of Bond Issuer in Indonesia using Euclidean Distance

Aulia Ikhsan[a*], Fikri C Permana[b,c], Ayu Nurulhaq Putri[d], Rifki Hamdani[e], Mukhtar Mukhtar[a], Syarif Abdullah[a], Rifqy Hafizh[f], Muhammad Hikam Adiguna[f], Dinda Dwi Anugrah Pertiwi[a], Kinanthi Trah Asmaraningtyas[a]

[a] *Department of Statistics, Universitas Sultan Ageng Tirtayasa, Jl. Jenderal Sudirman KM 3 Cilegon Banten*
[b] *Department of Sharia Banking Doctoral Program, UIN Syarif Hidayatullah*
[c] *KB Valbury Sekuritas, Jl. Jenderal Sudirman No. 86, Jakarta Pusat 10220, Jakarta, Indonesia*
[d] *Department of Applied Linguistics, Universitas Negeri Jakarta*
[e] *Department of Mathematics Education, Universitas Tidar*
[f] *Department of Agribusiness, Universitas Sultan Ageng Tirtayasa*
*Corresponding Author: aulia.ikhsan@untirta.ac.id

INFORMATION

Article information:
Submitted: 18 February 2025
Revised: 26 March 2025
Accepted: 31 March 2025
Available Online: 31 March 2025

Keywords:
Default; Investment Grade; Markov Chain; Euclidean Distance

ABSTRACT

The debt instrument (bond) as one of the investment instruments in the Capital Market has a main risk known as default. Default risk can be mitigated if investors assess the credit quality of the bond and its issuer, as measured by rating. In this research, the initial rating of issuers was investment grade (BBB or higher) and valid for at least 1 year, with their business operations based in Indonesia. The observation period was from 2007 to 2023. A Markov Chain was used to create a transition matrix to analyze transitions and defaults. The probability of AAA staying over 1 year is 0.9858 whereas the likelihood of AA, A, and BBB staying in the same rating is 0.9203, 0.8825, and 0.8630, respectively. The BBB in a 5-year transition has the highest probability of default by 0.0370. The Euclidean distance was used to measure the similarity of default durations. The 1-year and 3-year transitions have the shortest distance, at 0.00939. The conclusion of this research is that a higher rating has a higher probability of staying at the same rating and carries a lower risk. Furthermore, 1-year and 3-year transitions show similarities based on their probability of default.

INTRODUCTION

The corporate debt instrument, hereinafter referred to as a bond, is an investment instrument in the form of debt issued by a company containing the commitment of the issuer to repay the principal and its interest on the debt instrument (coupon) to the investor over a specified period of time [1]. The bond is one of the investment instruments in the capital market besides stocks. However, the bond and the stock have different characteristics. The stock represents ownership of a company. According to [2], stock investors have a chance to gain a higher return from the capital gain due to the liquidity of the stock but at the same

ISSN xxxxxx print/ xxxx-xxxx online © 2025
DOI: http://dx.doi.org/10.62870/tjs.v1i1.31335

time, the instrument has a higher risk because of the fluctuation of the stock price. Meanwhile, the bond has a stable return and a lower risk than the stock. Nevertheless, the bond still has an inherent risk.

The risk of the bond is the likelihood that investors do not get an appropriate return as expected or lose the capital over their investment in the bond [3]. The risk of the bond has various sources such as macroeconomics, business sector or industry, and issuer's business itself. Bond investments carry inherent risks, one of which is default risk.

Default risk or credit risk is the likelihood that the issuer is unable to meet its obligation to repay the principal or interest (coupon) of the bond it issued when it is due [1]. The authority of the capital market will require the issuer to be rated by one or more rating agencies before it issues the bond to mitigate such risk. The default risk is reflected by the issuer's rating. The rating is a forward opinion about the quality of the credit based on the financial condition of the issuer currently assessed [4]. If the issuer has a higher rating, then its bond rating is higher, and its risk is low, and vice versa. Therefore, the bond rating depends on the rating of the issuer.

The rating is the opinion about the capability of the issuer to meet its obligation to repay the principal and the coupon of the bond it issued at a specified time [5]. The rating of the issuer has some objectives in addition to representing the quality of the credit. The rating can be a reference for determining the coupon rate of the bond. If the rating is higher then the rate of the coupon will be lower. The rating of the issuer could be a reference for investors to invest in the bond.

There are eight common rating categories in Indonesia. According to [6], the ratings are AAA, AA, A, BBB, BB, B, CCC, and D. The AAA rating is the highest rating. It informs that the issuer has a superior capacity to meet its obligation. The AA rating is slightly different from the AAA rating, but the issuer in this rating has the strongest capacity to meet its obligations. The A rating informs that an issuer has a strong ability to meet its obligations, although its ability is still affected by adverse change of economic conditions. The BBB rating indicates that an issuer has an adequate capability to meet its obligation, although its capacity has a higher potency to be affected by adverse change of economic conditions. The BB rating indicates that the issuer has a somewhat weak capacity to meet its obligation and it faces continuous uncertainties or is affected by business, financial, and adverse economic conditions. The B rating indicates that the issuer has a weak capability to meet its obligations and its capacity may be weakened by business, financial, and adverse economic conditions. The CCC rating informs that the issuer is vulnerable to not meeting its obligations and it depends on favorable business and financial conditions to support the obligation. The D rating informs that the issuer is unable to meet its obligation. The AAA, AA, A, and BBB are categorized as investment-grade ratings, as they have low-moderate risk whereas the BB, B, and CCC are categorized, as speculative-grade and carry a high risk of default [7].

Ratings such as AA, A, BBB, BB, and B often include positive (+) or negative (-) signs to indicate relative strength within the category. As an example, A+ is interpreted as an issuer having a strong ability to meet its obligations but it is still affected by adverse changes in economic conditions, and the rating is higher than A and A-. Nevertheless, the rating is not always the same in the periods of time. The rating process is conducted every year as long as the issuer has an obligation related to its bond. The rating could change to higher, same, or lower. The change is documented in a transition matrix.

The transition matrix describes the probability of rating transition at a certain time [8]. In the transition matrix, the higher rating has a higher probability to stay in the same rating or to upgrade to a higher rating in a 1-year transition whereas the lower rating has a higher probability to downgrade to a lower rating or to default in a 1-year transition [9]. However [9] is limited in 1-year transition. Meanwhile, the bond issued has more than 1-year duration (tenor) so it is needed to research the transition rating of the issuer for more than 1 year.

Based on those backgrounds, this research aims to analyze the default pattern of investment grade rating in more than 1 year. According to [10], the bond with a longer tenor has a higher risk than the bond with a shorter tenor. Furthermore, this research also aims to determine the similarities between tenors based on the probability transition from investment grade to default.

RESEARCH METHODS

Source of Data

The data were collected from the Indonesia Central Securities Depository (KSEI) and Rating Agencies in Indonesia. The data are the rating transition of issuers in the period of 2007 – 2023. The issuers are the companies operating their business in Indonesia. Variables used in this research are Company Ticker, Company Name, Business Sector, Year (i), Rating in Year (i), Year ($i + 1$), Rating in Year ($i + 1$) (Rating ($i + 1$)), Year ($i + 3$), Rating in Year ($i + 3$) (Rating ($i + 3$)), Year ($i + 5$), and Rating in Year ($i + 5$) (Rating ($i + 5$)). The ticker and the names of some companies will be disguised to keep the confidentiality of the rating. Table 1 shows the sample data used in this research.

Table 1. The Sample Data of Rating Transition

Ticker	Company	Sec.	Year (i)	Rating (i)	Year ($i+1$)	Rating ($i+1$)	Year ($i+3$)	Rating ($i+3$)	Year ($i+5$)	Rating ($i+5$)
NBNP	PT Bank NBNP Tbk	F	2010	AA	2011	AA	2013	AA	2015	AA
NBNP	PT Bank NBNP Tbk	F	2011	AA	2012	AA	2014	AA	2016	AA
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
NBNP	PT Bank NBNP Tbk	F	2018	AA	2019	AA	2021	AA	2023	AA
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
KRND	PT Menjaga Uang (Persero)	F	2010	A	2011	A	2013	A	2015	A
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
KRND	PT Menjaga Uang (Persero)	F	2016	A	2017	A	2019	A	2021	AA
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
OCIR	PT Lezat Benar Selamanya	R	2017	BBB-	2018	BBB-	2020	BB+	2022	CCC
OCIR	PT Lezat Benar Selamanya	R	2018	BBB-	2019	BBB-	2021	BB+	2023	D
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

The sector is categorized into Financial Service (F) and Non-Financial Service (R). Companies such as banks, securities, and leasing are categorized as the Financial Services sector. Meanwhile, companies such as contractors, chemical producers, and consumer goods are grouped in the Non-Financial Services sector.

In the data processing, the rating with different signs will be considered an equal rating. For example, the AA+ and the AA- will be considered as the AA. The first step of data processing creates a transition matrix using the concept of Markov Chain. The 1-year, 3-year, and 5-year transition matrix will be created at this step. These durations of transition will be considered as objects and are considered as objects based on [9]. The second step of data processing creates default vectors. Every default vector contains the probability of default calculated from the transition matrix. The third step of data processing calculates the distance between two default vectors using Euclidean distance. Furthermore, the literature review will be presented below:

Matrix and Vector

A matrix is a data structure in the rectangular form containing data called entries [11]. The size of the matrix is determined by the total of rows and columns. Let $\boldsymbol{X}$ is the matrix with size $m \times n$, the matrix has m rows and n columns. The matrix $\boldsymbol{X}$ is written in the following way:

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

A vector is an arrow that has an initial and terminal point. The direction of the terminal point indicates the direction of the vector and the length of the arrow indicates the magnitude of the vector. Let $\boldsymbol{y}$ be a vector with n elements, then the vector $\boldsymbol{y}$ will be written as $\boldsymbol{y} = (y_1, y_2, \dots, y_n)$. Vector could be stated in the term of the matrix. A matrix with one row is called a row vector whereas a matrix with 1 column is called a column vector [11].

Markov Chain

One of the specific forms of the stochastic process is Markov Chain [12]. This process explains the probability that the state in the present will transition to another state in the future. The transition is affected by a state in the present, not in the past. Let Z_i is the stochastic process at the time i where $i = 0,1,2,\dots$; then $Z_i = s$ is the state of s at the time i. According to [13] P_{st} indicates the process will, when in state s, make a migration into state t. The formula to obtain P_{st} is written in the following way:

$$P_{st} = P(X_{i+1} = t \mid X_i = s, X_{i-1} = s_{i-1}, \dots, X_1 = s_1, X_0 = s_0) = P(X_{i+1} = t \mid X_i = s)$$

where 1) $0 \le P_{st} \le 1$ for $s \in \{1, 2, 3 \dots, m\}$ and $t \in \{1, 2, 3, \dots, n\}$
2) $\sum_{t=1}^{\infty} P_{st} = 1$ for all $s \in \{1, 2, 3, \dots, m\}$

To estimate P_{st}, the migration from state s to state t could be assumed as multinomial experiment. The estimation of the probability could be calculated using formula 1 [14].

$$\hat{P}_{st}(i) = \frac{n_{st}(i)}{n_s(i)} = \frac{\sum_{i=0}^{I} n_{st}(i)}{\sum_{i=0}^{I} n_s(i)}, \tag{1}$$

$n_s(i)$ is all observations or objects in state s at the time i whereas $n_{st}(i)$ is objects of $n_s(i)$ that migrates to t at the time $i + r$.

Distance

Distance is one of the important measurements in statistics. Distance is used to measure the closeness or similarity between two objects based on their characteristics or features. Outlier detection and discriminant analysis are two applications of distance [15]. Furthermore, the Kolmogorov-Smirnov test to assess the similarities between theoretical distribution and empirical distribution is another application of the distance [16].

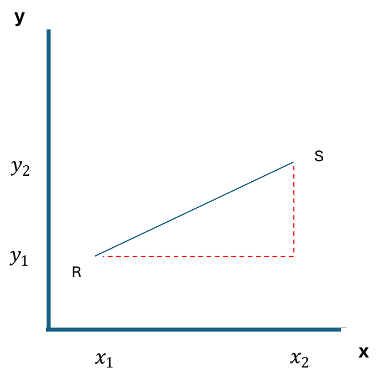

Figure 1. The distance between point R to point S

Let $R(x_1, y_1)$ and $S(x_2, y_2)$ in Figure 1 be two points on the coordinate system, then the distance between point R to point S or the length of point R to point S could be determined using Pythagoras theorem. The length of point R to point S or the distance between those points is the hypotenuse of a right triangle. The base of the triangle is calculated by subtracting x_2 and x_1 whereas the height of the triangle is calculated by substracting y_2 and y_1.. Therefore, the distance between point R to point S using Phytagoras theorem could be calculated as follows:

$$d(R,S) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (2)$$

The equation 2 is known as Euclidean distance. Based on that equation, the Euclidean distance could be expanded to more than two dimensions. Let $R(x_1, x_2, \ldots, x_p)$ and $S(y_1, y_2, \ldots, y_p)$ be two points on p dimensions, then the Euclidean distance between point R to point S could be calculated using equation 3.

$$d(R,S) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_p - y_p)^2} \quad (3)$$

RESULTS AND DISCUSSION

Tables 2 – 4 are 1-year, 3-year, and 5-year rating transition matrices. Rows of the matrix are the initial rating of the issuer and columns of the matrix are the transition of the rating. Elements of the matrix are the probability of transition. The probability is calculated by the formula in Equation 1. As an example, in 1-Year Rating Transition Matrix, the probability of AA rating migrating to AAA rating $\left(\hat{P}_{(AA)(AAA)}(i)\right)$ is calculated by dividing $n_{(AA)(AAA)}(i)$ by $n_{(AA)}(i)$. Here, $n_{(AA)}(i)$ represents the total number of observations with AA rating in the Rating (i) variable whereas $n_{(AA)(AAA)}(i)$ represents the total number of paired observations with AA rating in the Rating (i) variable and AAA rating in the Rating $(i+1)$ variable in sample data. Furthermore, in 3-Year Rating Transition Matrix, the calculation of $\hat{P}_{(AA)(AAA)}(i)$ follows the same formula but in this case, $n_{(AA)(AAA)}(i)$ represents the total number of paired observations with AA rating in the Rating (i) variable and AAA rating in the Rating $(i+3)$ variable in sample data.

Table 2. 1-Year Rating Transition Matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.9858	0.0142	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AA	0.0488	0.9203	0.0309	0.0000	0.0000	0.0000	0.0000	0.0000
A	0.0000	0.0544	0.8825	0.0491	0.0053	0.0000	0.0017	0.0070
BBB	0.0000	0.0046	0.0411	0.8630	0.0411	0.0046	0.0136	0.0320

Table 3. 3-Year Rating Transition Matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.9487	0.0513	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AA	0.1409	0.7870	0.0687	0.0034	0.0000	0.0000	0.0000	0.0000
A	0.0071	0.1283	0.7126	0.1164	0.0166	0.0000	0.0047	0.0143
BBB	0.0000	0.0174	0.1913	0.6521	0.0696	0.0087	0.0348	0.0261

Table 4. 5-Year Rating Transition Matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.9455	0.0545	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AA	0.2558	0.6419	0.0883	0.0093	0.0000	0.0000	0.0000	0.0047
A	0.0140	0.1754	0.6597	0.1088	0.0175	0.0000	0.0105	0.0141
BBB	0.0000	0.0741	0.2593	0.5370	0.0185	0.0000	0.0741	0.0370

The probability of AAA staying at the same rating in 1 year is highest among three durations. The probability is 0.9858. Meanwhile, the lowest probability is 0.0000. It spreads in some rating transitions, such as the transition from AAA and AA to B, as well as the transition from BBB to AAA.

For every initial rating on every duration, the highest probability occurs when the initial rating stays at the same rating or does not migrate to another rating. If those probabilities are compared, there is a descending pattern in 1-year and 3-year. The pattern shows that the higher initial rating has a higher probability of staying at the same rating, and vice versa. However, a different pattern is shown by the 5-year transition. During this duration, the AA rating has a lower probability of staying at the same rating than the A rating. Nevertheless, this probability is still lower than the probability of AAA and higher than the probability of BBB staying at the same rating. Furthermore, the longer duration has a lower probability for every initial rating to stay at the same rating. For example, the probability of A to stay at the same rating descends from 0.8825 in 1 year, 0.7126 in 3 years, and 0.6597 in 5 years.

The AA rating has a higher probability of upgrading to AAA than to downgrade to a lower rating for every duration. As an example, the probability of AA upgrading to the AAA rating in the 3-year transition is 0.1409 whereas the probability of AA to downgrade to a lower rating is 0.0721. However, a different result is shown in 1-year and 3-year transitions. The probability of A rating upgrading to higher ratings on those two durations is lower than the rating to downgrade to lower ratings. Meanwhile, the probability of BBB upgrading to higher ratings is higher than the rating to downgrade to lower ratings in the 3-year and the 5-year transition. Nevertheless, AA, A, and BBB have a higher probability of upgrading to higher ratings in a 5-year transition.

The probability of default and rating grade have an opposite direction. The lower rating has a higher probability of default, and vice versa. The direction is shown in Figure 2. The data in Figure 2 came from column D in every transition matrix. The BBB rating in the 5-year transition has the highest probability by 0.0370. However, the BBB rating in the 3-year transition has the lowest probability compared to the same rating in the 1-year and the 5-year transition. The A rating in the 1-year transition has the lowest probability compared to the same rating in the 3-year and the 5-year transition. The AA rating in 5-year has the highest probability compared to the same rating in 1-year and 3-year.

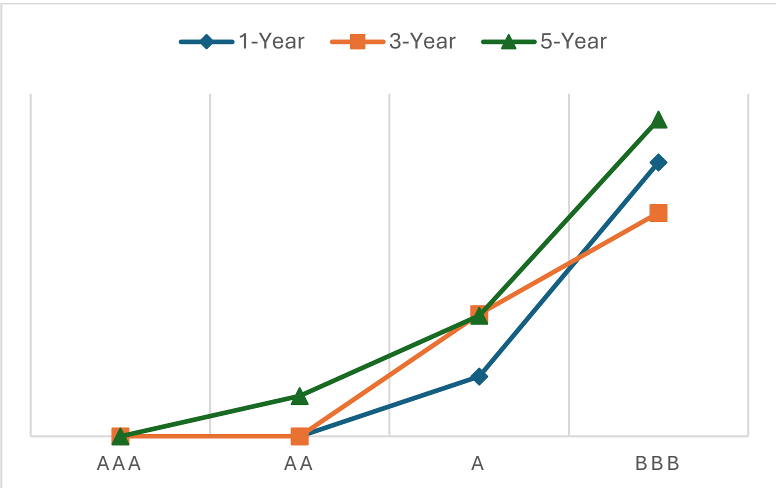

Figure 2. The Probability of Default

From the previous explanation, there is no similarity is shown by a pair of transition. As an example, in the AA rating, the probability of default is similar between 1-year and 3-year. Meanwhile, in the A rating, the probability of default is similar between 3-year and 5-year. Therefore, Euclidean distance in equation 3 will be used to determine a pair of transition that has a similarity based on the probability of default. Every time-transition will be a point or a vector. Elements of the vector are the probability of default of the AAA, AA, A, and BBB rating. Let $r = (0.0000, 0.0000, 0.0070, 0.0320)$ is the default vector of 1-year transition, $s = (0.0000, 0.0000, 0.0143, 0.0261)$ is the default vector of 3-year transition, and $t = (0.0000, 0.0047, 0.0141, 0.0370)$ is the default vector of 5-year transition; then the Euclidean distance between time-transition is presented in Table 5.

Table 5. Euclidean Distance between time-transition

Transition	Euclidean Distance
1-Year and 3-Year	$d(r, s) = 0.00939$
1-Year and 5-Year	$d(r, t) = 0.00987$
3-Year and 5-Year	$d(s, t) = 0.01187$

The probability of default for investment-grade ratings is similar between 1-year and 3-year transitions. This is based on the Euclidean distance of the time-transition pair, which has a smaller value compared to other pairs, specifically 0.00939. The period between 1-year and 3-year indicates a significantly lower likelihood of rating changes than the 1-year and 5-year transition, which has a Euclidean distance of 0.00987, or the 3-year and 5-year transition, which stands at 0.01187. This value confirms that investment-grade ratings exhibit a comparable probability of default in both the 1-year and 3-year transitions.

CONCLUSION

The transition matrix, whose elements obtained from the concept of Markov Chain, provides information about the issuer's rating. Based on the rating data of the issuer, the higher rating has a higher probability of staying at the same rating or consistent for 1 year and 3 years. However, this consistency differs for a 5-year transition; yet the investment grade ratings have a higher probability of upgrading to a higher rating in this time transition than the probability of downgrading. The probability of AA rating to upgrade is the highest for all time-transition. The probability of default and rating grade have an opposite direction in other words the lower rating has a higher probability of default. The probability of default is similar between the 1-year and 3-year transition. This indicates that bond investments in an issuer with an investment-grade rating for a one-year or three-year period carry a similar level of risk, which is significantly lower compared to other rating categories and time frames.

REFERENCES

[1] F.J. Fabozzi, and F.A. Fabozzi, *Bond Market, Analysis, and Strategies*. Massachusetts: The MIT Press, 2021.

[2] Syamsudin, and M. Khaddafi, "Analisis Komparatif Risiko dan return Pada Saham dan Obligasi di Bursa Efek Indonesia: Implikasi bagi Investor", Jurnal Ekonomi, Bisnis, dan Manajemen, vol. 3, no. 2, pp. 61-76, Jun. 24, doi: https://doi.org/10.58192/ebismen.v2i3.2209.

[3] T. Lei, "Research on Bond Market in Risk Management", Highlights in Business, Economics, and Management, vol. 41, pp. 414-419, Oct. 24, doi: 10.54097/cdqpfr79.

[4] A. Nugraha, "Analisis Pengaruh Perubahan Bond Rating terhadap Abnormal Return untuk Saham yang Terdaftar di Bursa Efek Indonesia", Universitas Indonesia, 2010.

[5] OJK, "Peraturan Otoritas Jasa Keuangan Republik Indonesia Nomor 49/POJK.04/2020 Tentang Pemeringkatan Efek Bersifat Utang Dan/Atau Sukuk", Lembaran Negara Republik Indonesia, no. 273, 2020.

[6] PEFINDO, "Rating Definition", 2025. https://www.pefindo.com/services.

[7] M. A. Aslam, "Does the Percentage of Investment Grades Given by Rating Agencies Impact their Market Share?", Financial Markets, Institutions, and Risks, vol. 4, no.1, pp. 5-31, Apr. 20,

doi: http://doi.org/10.21272/fmir.4(1).5-31.2020.

[8] M. D. Hadad, W. Santoso, B. Santoso, D. Besar, and I. Rulina, "Rating Migration Matrices: Empirical Evidence in Indonesia", IFC Buletin, no. 31, Apr. 20.

[9] PEFINDO, "The Default Study of Corporate and Corporate Debt Securities Rated by PEFINDO", 2023. https://www.pefindo.com/default-study.

[10] S. Rahardjo, *Panduan Investasi Obligasi*. Jakarta: PT Gramedia Pustaka Utama, 2004.

[11] H. Anton, and C. Rorres, *Elementary Linear Algebra.* USA: Wiley, 2014.

[12] F. N. Masuku, Y. A. R. Langi, and C. Mongi, "Analisisi Rantai Markov untuk Memprediksi Perpindahan Konsumen Maskapai Penerbangan Rute Manado-Jakarta", Jurnal Ilmiah Sains, vol. 18, no. 2, pp. 75-79, Oct. 18.

[13] S. M. Ross, *Introduction to Probability Models.* Burlington: Elsevier, 2007.

[14] J. H. E. Christensen, E. Hansen, and D. Lando, "Confidence Sets for Continous-Time Rating Transition Probabilities", Journal of Banking & Finance, vol. 28, no. 11, pp. 2575-2602, Aug 04, doi: 10.1016/j.jbankfin.2004.06.003.

[15] G. M. Venturini, "Statistical Distance and Probability Metrics for Multivariate Data, Ensembles and Probability Distribution", Department of Statistics Universidad Carlos III De Madrid, 2015.

[16] C. P. Kitsos, and C-S. Nisiotis, "Considering Distance Measures in Statistics", Biometrical Letters, vol. 59, no.1, pp. 65-75, Jun. 22, doi: 10.2478/bile-2022-0006.